Modelling longevity risk using Lee-Carter model

Gitau, James King'ara 063292

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Strathmore Institute of Mathematical Sciences (SIMS)
Strathmore University
Nairobi, Kenya

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Declaration

I declare that this work has not been previously submitted and approved for the award of a degree by this or any other University. To the best of my knowledge and belief, the Research Proposal contains no material previously published or written by another person except where due reference is made in the Research Proposal itself.

Gitau, James Kinyua .... [*Name of Candidate*]
.... [*Signature*]
January 23, 2017 .... [*Date*]

This Research Proposal has been submitted for examination with my approval as the Supervisor.

Mary Wanza Kiplimo .... [*Name of Supervisor*]
.... [*Signature*]
23/1/17 .... [*Date*]

Strathmore Institute of Mathematical Sciences (SIMS)
Strathmore University.

Abstract

Longevity risk is one of the remaining frontiers challenging modern financial markets and financial engineering. It is a major policy issue for governments around the world driven by the increase in the proportion of the aged resulting from improved healthcare and thus improved mortality rates. Increasing and uncertain longevity has emerged as a key risk affecting individuals, pension schemes, insurers and governments in both the developed and emerging world. This paper discusses longevity risk in detail, and its significance to the modern world.

Keywords; Longevity risk, Lee-Carter model.

Contents

Introduction

Population projections are estimates of the population for future dates. They are typically based on an estimated population consistent with the most recent decennial census and are produced using the cohort-component method. Over the past 10 years there has been an increasing trend in the growth rate of the population and a significant decrease in the crude death rate. This is because of improved lifestyles of Kenyans and better health conditions.

Now, diseases such as HIV/AIDS[1] and cancer are well managed due to measures that the government has put in place to reduce rates of infection and to take care of patients so that they can live longer. The government aims at reducing new HIV infections by 75%, reduce HIV related mortalities by 25% and increase domestic financing of the HIV response to 50% (National aids control council, 2014).

Cancer, which is the third highest cause of death after infectious diseases and cardiovascular diseases, contributes approximately 7% of all the deaths in Kenya (Ministry of public health and sanitation and Ministry of medical services, 2011). This is however expected to reduce due to prevention and management measures being taken by the Kenyan government. The Kenyan government is also running campaigns against unhealthy living such as smoking, excessive consumption of alcohol and other forms of drug abuse thus improved mortality.

The purpose of this report is to model uncertainty about the future based on mortality and life expectancy while understanding longevity trends in the life insurance markets and the pension sector. Longevity is the risk of underestimating the survival probability leading to inappropriate premiums; being risk derived from a future mortality rate which ex-post reflects not the forecasted on, (Brouhns, Denuit, & Vermunt, 2002).Currently we are trying to forecast future population using a mathematical model based on a pre-existing data in contrast with the intercensal estimates and censuses which usually involve some sort of field gathering. Population projection involves considering many assumptions.

These assumptions are based on fertility, mortality and migration within the population. Other factors that were also considered while modeling population projection were; gender, age,

[1] Human Immunodeficiency Virus. If left untreated, HIV can lead to the disease AIDS (Acquired Immunodeficiency Syndrome).

period and cohort. Life expectancy at birth has increased globally from 65 years in 1990-1995 to 70 years in 2010-2015.Hence the overall rate of future mortality has shown significant improvement for the last three decades though it could decline in the future. But looking at the government vision of 2030 and second medium term plan (2014-2018) they both aim to better the healthcare.

The Kenya health policy (2014-2030) indicates that 16% improvement of life expectancy, 50% reduction in annual mortality and 25% reduction in time spent in ill-health. Usually, we view such mortality improvements from an optimistic perspective. From a general point of view, there is an average survival improvement i.e. increased life expectancy in developed countries including European countries, New Zealand; Australia and in African countries. Per the report by the World Health Organization, in 2014 in Kenya; mortality projections have been pointing to increased lifespan.

Although there are wide disparities such that the average length of life in Africa in 2010-2015, is 12 years shorter than the global average and 21 years shorter than North America. These changes in the mortality trend affect the pricing and reserve allocation for life annuities from a risk profile point of view. In the UK, Biffs and Blake reported that for every additional year of life expectancy of 65 is estimated to add at least 3% of the present value of pension liabilities. Therefore, there is need to consider measures to mitigate the adverse effects on pension providers and insurers while still guaranteeing stable and secure benefits throughout retirement and other insurance benefits to policyholders. Updated mortality tables are no reprieve since they do not take changing trends into account.

The results of high expectancy are that there will be an increase in the number of elderly people in Kenya. Retired Kenyans are living longer as compared to the past 10 years. The U.S Department of Commerce- Bureau of the Census predicts that by the year 2020, the older population (above age 65 years) will be growing faster than the total population. They further project that the growth rate of the Kenyan population above 65 years will be 5% per year by 2020. This is significant compared to the projected population growth rate of 3.2%. This automatically indicates that retired Kenyans will be receiving pensions for way longer than the pension schemes had estimated. The risk of longevity is simply a few years away from becoming a reality.

The burden of paying pensions has been quite heavy for the government of Kenya mainly due to insufficient funds. In 2009, the retirement age in Kenya was increased from 55 to 60 years to try and postpone payment dates of pensions. This would buy the government more time to gather funds to pay retirees in the civil service pension scheme. The civil service pension scheme has about 400,000 members. 53% of the pension are teachers, 30% are employees of different ministries in the government while 17% are in the disciplined forces. 21% of the total members are already receiving pensions.

Taking everything constant without upward adjustment of retirement age, the rate of growth of pension payment is projected to increase to some insurmountable amounts not affordable to the government (Retirement Benefits Authority). Even at present, the civil service scheme does not dutifully meet pension obligation in time. How worse will it be when there will be more pensioners?

Epidemiologic transition also forms a basis for my research (underlying improvements in longevity that occur with the demographic transition). Demographic transition is when population funds to shift overtime from being characterized by high fertility and mortality to low fertility and mortality. This transition is a fundamental component of development (Chant & Dyson, 2010).

Epidemiology transition is characterized by initial decline in death rates due to communicable diseases in the early stages of the transition followed by subsequent reductions in mortality attributable to non-communicable diseases in the advanced stages (United Nations, 2012)

To address the emerging longevity risks, various stochastic methods have been developed. They include; McNown & Rogers, (1992), Bell & Monsell, (1991), Lee and Carter (1992). In our project, we are using the Lee Carter model, developed by Lee and Carter specifically for the U.S population. It is the most profound model applied in many countries. The Lee carter model is a numerical algorithm used in mortality forecasting and life expectancy forecasting. It models age specific mortality rates based on cohort component. Assumes that the mortality trends are ruled by a single parameter called mortality index. It works in that the input to the model is a matrix of age specific mortality rates (years in rows and ages in columns) and the output is the forecasted mortality rates.

- Since the introduction of the Lee-Carter model, it has been widely used based on the stochastic forecasts of the finances of the U.S social security system and other aspects of the U.S federal budget office.
- An advantage of the Lee-Carter model compared to other stochastic models is that it uses a small number of parameters. It's Easy to implement and outperforms other models with respect to errors. It can also be extended to obtain broad interpretation.

There are however challenges in that the future development of life expectancy is uncertain. Also, the stringent model has been known to produce narrow confidence intervals resulting in the underestimation of the risk.

The effects of the trends are that they cost significant challenges to the government as well as pension funds and life insurers. Current developments in using the lee carter model are the smoothing method, which reduces the number of parameters used.

Problem statement

Longevity risk has majorly been an alarming issue in the developed countries. Many underdeveloped nations have not experienced the risk because their mortality rates are still higher than the rates in other countries of the world. Kenya though, has one of the fastest growing economies in Africa which has resulted in better livelihood, a constantly decreasing mortality rate and overall good standards of living way above that of third world countries. Good management of HIV/AIDS in the country has also been a factor that has lowered mortality rate in Kenya.

The current trend is that of an increasing life expectancy in the country despite a few factors like attacks by terrorists and post-election violence which lead to very high numbers of death. This trend is continuing because of the constant economic, political and social developments in the country. Life expectancy may even shoot higher if the planned "Vision 2030" is completely fulfilled because that will transform the country to the level of developed nations.

The ratio of life expectancy in 2005 to that of 2015 is 1.17 i.e., life expectancy has grown by approximately 17 % within 10 years. If this ratio is applied to the future, the life expectancy will be at 75 years in 2025 and 85 years in 2035. The current retirement age in Kenya is 60 years old meaning that by 2025, pension funds will be paying retirees for about 15 years and by 2035, the burden will be heavier by an additional 10 years. Clearly, longevity risk will soon catch up with life offices in Kenya and if not handled well in good time could result to a financial crisis for offices that issue pensions.

Kenyan actuaries are already raising the concern because the danger is already becoming evident. In fact, some Kenyan actuaries refer to longevity risk as an already ticking time bomb waiting to explode in good time. Furthermore, the life tables that are currently in use in the country are quite old and do not depict the current mortality situation in the country. Longevity risk is already a risk in Kenya and it is only a matter of time before it can become a menace. There is an urgent need to strategize on ways to handle this risk lest it will be too late in the coming 10 years. Pension schemes in Kenya need a good plan to handle longevity risk.

Research objectives

Analyze longevity risk and its significance.

Quantify the significance of longevity risk.

Literature review

Given the significant improvements in life expectancy, longevity risk stands as a ticking time bomb. It's therefore prudent to be cognizant of this risk and put in place structures to mitigate it. According to the Retirement Benefits Authority the total industry assets stand at KES 814.11 billion as of December 31, 2015. This is the amount of money exposed to longevity risk hence the need to try and hedge this fundamental risk.

In developing countries improved life expectancy is hinged majorly on improved child mortality rates and to a lower extent the level of infrastructure in the economy. Wide disparities still exist such that average length of life in Africa in 2010 -2015 is shorter than the global average by 12 years. Median probabilities of survival to age 5 increased from 844 per 1000 live births in 1990-1995 to 908 per 1000 live births in 2010-2015 (United Nations, department of ecomonic and social affairs, population division, 2013).

Research conducted by Continuous Mortality Investigation in the UK found out that life expectancy at age 65 for males in England and Wales has risen dramatically by six years between 1970 and 2013, in contrast to a rise of around one year between 1841 and 1970 (Shkolnikov, Barbieri, & Wilmoth, 2015). This has in turn led to massive effects on pensions and life annuities. With pension liabilities of the UK private sector defined benefit pension schemes estimated at £2 trillion (Robertson, 2015), it will cost these schemes £60 billion if their pensioners are to live on average one year longer than assumed.

However, preventive measures are being taken to avoid this loss. Many pension schemes are transferring the risk to big insurance companies who are willing to take on the risk e.g. Prudential Insurance Company. Prudential Insurance Company then reinsured the longevity risk transferred been the largest longevity risk transfer in the world so far.

Models used to model mortality are as many as they are varied. They include; p-spline model introduced by Currie, Durbin and Eilers, Cairns Dowd and Blake model, Haberman and Renshaw method and the Booth-Main Donald-Smith variant.

The Booth-Main Donald-Smith variant has an advantage over the Lee-Carter model in that it's assumed that k_t is linear and jump off rates are taken to be the fitted rates based on fitting methodology. Its disadvantage is that k_t adjustment involves fitting to exponential age distribution.

An advantage of p-spline regression is that calculations can be easily implemented using a data augmentation algorithm. It is used to smooth historical death rates simultaneously across age and year. During forecasting, model fitting is done at the same time by treating the forecast period as missing data. The method fills in the missing values by extending the smooth surface fitted on the historic data to the fitting period. Researchers with the Continuous Mortality Investigation (CMI), wrote a number of working papers about investigations into the use of p-splines for creating mortality forecasts (Mortality committee, 2005).The main limiting factor they found is that mortality forecasts are relatively sensitive to adding another year of data to the end of the fitting period.

The Cairns Blake and Dowd method advocates for building of smoothness of mortality rates across adjacent ages in the same year. Currie Durbin and Eilers method assumes smoothness across both ages and years.

The Renshaw-Haberman model is an extension of the lee-carter with an extra parameter. This extra parameter is a random cohort effect that's a function of the year of birth.

The above models are tedious and cumbersome to use and its difference from other well used models (Lee-carter model) is negligible or can be adjusted accordingly to fit the ideal model.

Booth, Hyndman, Tickle, & De Jong, (2006) compare the short-term to medium-term accuracy of five variants or extensions of the Lee and carter method. Lee Carter model sought to model and forecast the time series of the mortality of United States of America (Lee & Carter, 1992). The model is simple and easier to understand compared to P-spline and CDB methods hence more useful to this study. However, the shortcoming of the Lee-Carter model has been that it assumes that the ratio of the rates of mortality change at different ages remains constant over time whereas evidence of substantial age-time interaction has been found.

Rotation is said to occur when mortality decline is decelerating at younger ages and accelerating at old ages. This is expected to occur in developing countries as they attain high life expectancies. It is difficult to handle rotation especial in models that include all age groups.

In low-mortality countries, mortality decline is decelerating at younger ages and accelerating at older ages, corresponding to a rotation of the b_X in the Lee-Carter method. This rotation is expected to appear also in developing countries, when infant and child mortality drop to low levels and when resources for reducing old age mortality become increasingly available.

But this rotation, even in low-mortality countries, is still too subtle to model and to project in ways that are entirely data driven and include all ages. Without modelling this rotation, however, long-term projections will show anomalous results. Normally, a mortality curve will be U-shaped, with relatively high infant mortality, minimum in the teens, and rising mortality thereafter. The forecast method we propose in this paper incorporates these by assuming that the b_X coefficients from ages 0 to 65 converge to a single constant value. Above age 65 or 70 the b_X coefficients decline with age in any event, guaranteeing that mortality will continue to rise steadily with age.

A subtle trend of historical mortality change among the low-mortality countries, is that the decline of infant and child mortality was decelerating, and the reduction of old-age mortality was accelerating (Horiuchi and Wilmoth 1995; Kannisto et al. 1994; Li and Gerland 2011). This trend implies that the age-specific rates of mortality-decline are rotating over time. This change in b_X is not utilized by any existing Lee-Carter approaches as far as we know. Without modelling these changes, however, longer-term projections (50 to 100 years or longer) could imply questionable age patterns of mortality, especially for low-mortality countries where the subtle trends are more significant.

To avoid this, Li and Gerland (2011) introduced a robust rotation in the Lee-Carter b_X and called it the Lee-Carter method with robust rotation, which is subjective and may entail unnecessarily strong modifications. In this paper, we improve the rotation model by providing a more objective basis for it and by making the rotation occur continuously over time rather than abruptly. We call the proposed method the Lee-Carter method extended to model the rotation.

In recent longevity studies, other successors of Lee-Carter model have been used. They include the smoothing model which reduces the number of parameters used. Renshaw & Haberman (2008) is also another model used. It incorporates a cohort parameter to model variations in mortality among different individuals in the cohort. This model involves heavy data demand but with the recent use of hi-tech programs this is not an issue anymore.

Methodology

Assumptions

- The data used is correct and free of bias.
- Crude Death rates behave in the same way as central death rates.
- Grouping of data into five-year age groups has eliminated any misrepresentation of ages.
- Any error terms as found in the model are independent.
- Most developing countries have similar mortality trends especially those in the same geographical regions.

Limitations

- Future developments in life expectancy are uncertain.
- The forecast does not reflect uncertainty as to whether the model used is appropriate for the data used.
- Past trends may not be indicative of future trends. The method assumes a constant change in the pattern of mortality, when in fact the rates of decline in mortality may change.

Demographic transitions

To explain the methodology (Lee-Carter model) some demographical concepts are essential:

Age-Specific Death Rates

This the ratio of the number of deaths within a specified age group in each geographic area during a certain period to the corresponding population at risk of the same age group, in the same geographic area during the specified time of study. If the number of deaths is obtained through vital statistics, the denominator is estimated based on census data (Ortega, 1987).

Life Expectancy at Birth

Age-specific life expectancy is an estimation of the average number of the remaining years that a person would be expected to live if current mortality conditions were constant. It is calculated considering the age-specific death rates. Its formula is $e_x = {}^{T_x}/_{l_x}$ where T_x is total person years lived by the cohorts and l_x is the number of cohorts under study at time x.

Data

A sample that represents the trend in most countries was picked by comparing mortality data from developing countries both in Africa and Europe. The list was based on the UN member States (G77) and the list of International Standards Organization (ISO) members in the UN Eastern Europe Group of Countries. The sample data includes the mortality rates of males and females over a period of 20 years. A sample of projected mortality tables being used in developing countries from all over the world was taken. These mortality rates are later be compared to the rates that we estimate using the lee-carter model. The sample mortality tables used are found in Life tables for 191 countries: data methods and results. (World Health Organization)

The sample mortality data for developing countries is obtained from Human Mortality Database.

Lee-Carter model

The model Lee and Carter suggested in 1992 was $\log q_x(t) = a_x + b_x k_t + \varepsilon_x(t)$ where;

> $q_x(t)$ Is the probability of death at age x in year t.
> a_x Is the average of $\log q_x(t)$ over time t which describes the general pattern of mortality by age.
> k_t Is the time trend for the general mortality.
> b_x Indicates the sensitivity of $\log q_x(t)$ at age x as the k_t varies.
> $\varepsilon_x(t)$ Is the residual term at age x and time t.

The estimation of b_x and k_t cannot be solved explicitly and the model cannot be fit with ordinary regression methods. In the original paper, Lee & Carter (1992) used the singular value decomposition (SVD) method to find a least squares solution.

The extended Lee-Carter method

In this section, we suggest a strategy of initially projecting life expectancy using the standard LC, and then finding the adjusted values $K(t)$ to fit the projected life expectancy using the rotational $B(x,t)$. Thus, introducing the rotation does not change the projected values of life expectancy, but rather just changes the age pattern of mortality that generates those projected values (Li, Lee, & Gerland, 2013).

Knowing the values of $b_u(x)$, e_o^u, and $B(x,t)$, the projected $e_o(t)$ by sex can be fitted by finding a value of $K(t)$, which will differ from that of the Lee-Carter $k(t)$. Thus, the Lee-Carter method is extended to;

$$\log[m_{m/f}(x,t) = a_{m/f}(x) + B(x,t)K_{m/f}(t)$$

Where subscript f or m refers to female or male, respectively.

The extended Lee-Carter method reduces to the Lee-Carter method when the projected $\epsilon_0(t)$ is smaller than 80 years. When the projected $\epsilon_0(t)$ exceeds 80 years, the extended Lee-Carter model will gradually depart from the Lee-Carter model, as the decline of death rates at younger ages decelerates and at older ages accelerates.

Weighted least squares

To achieve a unique solution the following restrictions are used,

$\sum_{x\,min}^{x-m} b_x = 1$ (a)

$\sum_{t\,min}^{t-n} k_t = 0$ (b)

These restrictions make no modifications to the model. If we replace b_x with $b_x *= {}^{c}/_{b_x}$ then k_t could be replaced by $b_x *= {}^{k_t}/_{c}$ and the model is unchanged.

To estimate the parameters, we choose the values that minimize Q;

$$Q = \sum_{x,t} (a_x + b_x k_t - m_{xt})^2$$

Where $m_{xt} = \log q_x(t)$ and Q is subject to the constraints (a) and (b) above.

To find values that minimize Q we introduce Lagrange's multipliers; α and β and minimize

$$R = Q - \alpha \sum_t k_t - \beta \sum_x {b_x}^2$$

Thus, take the derivative of R in respect of a_x, b_x and k_t respectively;

$^{\partial R}/_{\partial k_t} = 2\sum_x b_x(a_x + b_x k_t - m_{xt}) - \alpha$ for every t.

$^{\partial R}/_{\partial b_x} = 2\sum_t k(a_x + b_x k_t - m_{xt}) - 2\beta b_x$ for every x.

The derivatives are set equal to zero;

$^{\partial R}/_{\partial a_x} = 0, ^{\partial R}/_{\partial b_x} = 0$ and $^{\partial R}/_{\partial k_x} = 0$

Solving the first equation,

$$2\sum_t (a_x + b_x k_t - m_{xt}) = 0$$

$$na_x + b_x \sum_t k_t - \sum_t m_{xt} = 0$$

$$\sum_{t\,min}^{t-n} k_t = 0$$

$$na_x - \sum_t m_{xt} = 0$$

$$a_x = {}^1/_n \sum_t m_{xt} \qquad (1)$$

The estimate for a_x is thus computed as the average over time of the logarithm of the central death, which corresponds to the definition.

Now define $z_{xt} = m_{xt} - a_x, \sum_t z_{xt} = 0$ for every x.

Now rewrite the second equation;

$$2\sum_x b_x(a_x + b_x k_t - m_{xt}) - \alpha = {}^{\partial R}/_{\partial k_t} = 2\sum_x b_x(b_x k_t - z_{xt}) - \alpha$$

If you set the new expression of the derivative about k_t equal to zero (0) we have

$$2\sum_x b_x(b_x k_t - z_{xt}) - \alpha = 0$$

$$2\sum_x b_x(b_x k_t - z_{xt}) - {}^{\alpha}/_2$$

$$k_t \sum_x b_x{}^2 - \sum_x b_x z_{xt} = {}^{\alpha}/_2 \qquad (\sum_{x\,min}^{x-m} b_x{}^2 = 1)$$

$$k_t - \sum_x b_x z_{xt} = {}^{\alpha}/_2 \qquad (2)$$

Taking the sum over t in the equation (2) above you get;

$$\sum_t (k_t - \sum_x b_x z_{xt}) = \sum_t {}^{\alpha}/_2$$

$\sum_t k_t - \sum_t \sum_x b_x z_{xt} = \sum_t {}^{\alpha}/_2 \quad \sum_{t\,min}^{t-n} k_t = 0$

$m \sum_x b_x z_{xt} = m\, {}^{\alpha}/_2 \quad (\sum_t z_{xt} = 0) \quad \alpha = 0$

Now you have an expression for k_t by putting $\alpha = 0$ in equation (2). Thus;

$$k_t = \sum_x b_x z_{xt} \text{ for every } t. \qquad (3)$$

The constraint for k_t is now fulfilled. If you use the expression z_{xt} in the third and last equation;

$$2\sum_t k(a_x + b_x k_t - m_{xt}) - 2\beta b_x = 2\sum_x k_t(k_t b_x - z_{xt}) - 2\beta b_x$$

and set this to equal zero (0) we have;

$$2\sum k_t(k_t b_x - z_{xt}) - 2\beta b_x = 0$$

$$b_x(\sum_t {k_t}^2 - \beta) = \sum_t k_t z_{xt} \text{ for every } x. \qquad (4)$$

Take the square of both sides and summarise over x to get

$$\sum_x b_x \sum_t ({k_t}^2 - \beta)^2 = \sum_x \left(\sum_t k z_{xt}\right)^2$$

$(\sum_t {k_t}^2 - \beta)^2 \sum_x (b_x)^2 = \sum_x (\sum_t k_t z_{xt})^2 \qquad (\sum_{x\,min}^{x=m} {b_x}^2 = 1)$

$$\left(\sum_t {k_t}^2 - \beta\right)^2 = \sum_x \left(\sum_t k_t z_{xt}\right)^2$$

$$b_x \sum_t {k_t}^2 - \beta = \sqrt{\sum_x \left(\sum_t k_t z_{xt}\right)^2}$$

Now to the get the expression for b_x, insert β's equation in (4);

$$b_x\left(\sum_t k_t^2 - \beta\right) = \sum_t k_t z_{xt}$$

$$\left(\beta = \sum_t k_t^2 - \sqrt{\sum_x\left(\sum_t k_t z_{xt}\right)^2}\right)$$

$$b_x\left(\sum_t k_t^2 - \sum_t k_t^2 + \sqrt{\sum_x\left(\sum_t k_t z_{xt}\right)^2}\right) = \sum_t k_t z_{xt}$$

$$b_x = \sum_t k_t Z_{xt} \Big/ \sqrt{\sum_x (k_t z_{xt})^2} \text{ for every } x \qquad (5)$$

The estimation of a_x is easily calculated from the observed one year death probabilities. The equations for estimating b_x and k_t; (3) and (5) are complicated to solve explicitly, but it is possible to find a solution with a rather low number of iterations. To get a starting value for b_x, we assume that b_x is independent of x, and equal to $1/\sqrt{m}$, where m is the number of ages or age groups.

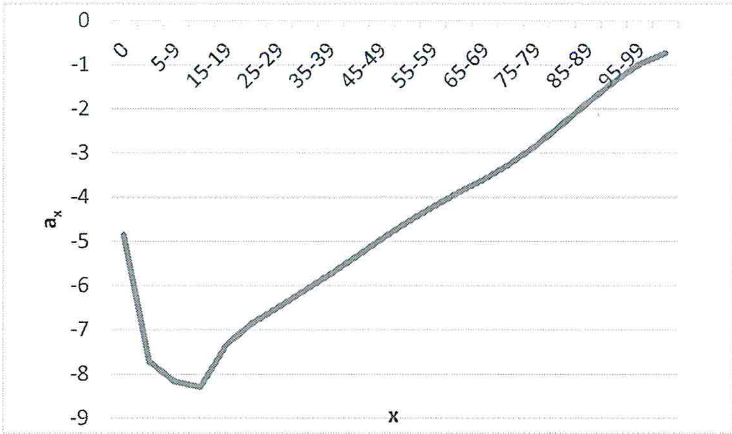
In the original paper written by Lee and Carter they made a re-estimation of k_t to get the observed number of deaths equal to the fitted number of deaths, i.e. $D_t = \sum_x exp(a_x + b_x k_t) N_{x,t}$

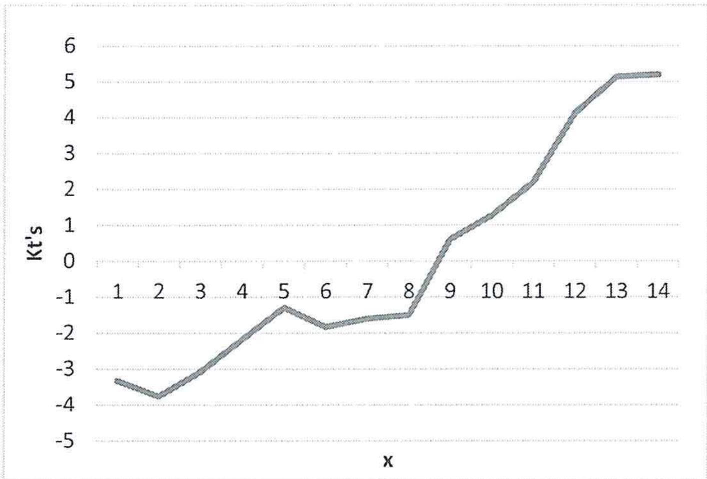
Here D_t is the total number of deaths in year t and $N_{x,t}$ is the population of age x in year t. No analytic solution is available so it can only be done by searching over a range of value of k. However, this second stage of estimation does not have any impact on whether the Lee-Carter model could be fitted to the data or not.

Data analysis

In this section, the methodology outlined is applied to the actual data by use of Microsoft excel. Intervals from the year 2000 are used since it's from around that time that improving mortality rate were recorded as shown in the appendix.

First find the base logarithms of all ages across all years. Subsequently, find the average of the logarithms for each age across all the years, a_x's (general mortality by age).



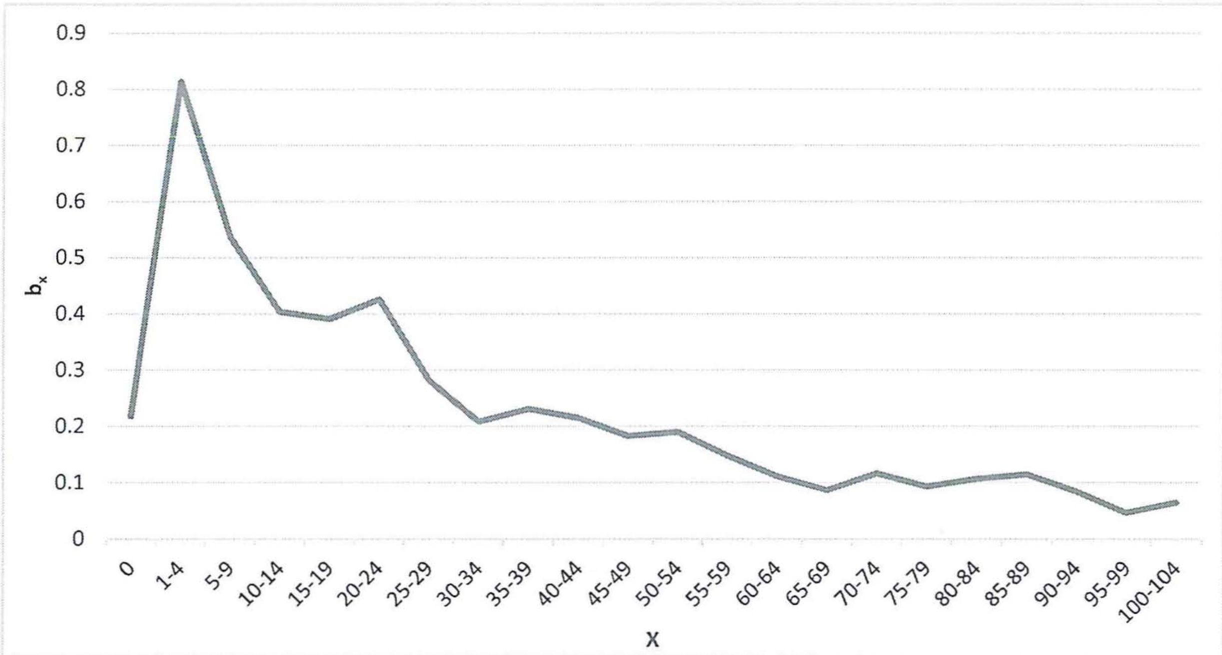


A graph of a_x against x

The results show that a_x values are increasing with age. This implies an upward trend in mortality with respect to the age groups. Younger ages have a lower mortality rate than the older ages. The reason for this disparity is probably due to differences in exposure to risk e.g. family responsibilities, stress, and age related health issues. The ages one (1) to four (4) behave differently as these ages are characterized by high mortality rates due to factors such poor post-natal care &c.

Then find $Z_{x,t}$ by subtracting the a_x from the natural logs. $Z_{x,t}$ has been constrained such that its summation is equal to zero.

Subsequently, find $k_{x,t}$ using the approximation formula $k_{x,t} = \sum Z_{x,t}$ for all time periods.



A graph of K_t's against x

The constraint of b_x values that $\sum b_{x^2} = 1$ is satisfied. This provides the sensitivity of $\log q_x(t)$ at age x as k_t varies.

Parameter b_x describes the tendency of the mortality at age x to change as the general level of mortality a_x changes. This indicates that when b_x is large for some x, the death rate at age x varies a lot than the general level of mortality change and when its small, then the death rate at that age varies a little. The b_x for ages between zero (0) and twenty-four (24) are generally higher than the rest of population. This can be attributed to high child mortality rates and high teenage fatalities due to experimentations and car accidents that come with high teenage adrenaline.

A graph of b_x against x

Forecasting future mortality rates is done by computing only the projected k_t's since we already have the other parameters; a_x and b_x, constant for every age. Lee and Carter predicted the mortality index in their original paper using a standard univariate time series model ARIMA (0,1,0). It demonstrated that other ARIMA models might be preferable for different data sets, but in practice the random walk with drift model (RWD) for k_t has been used almost exclusively.

Projected k_t's is found by; $k_t = k_{t-1} + \theta + \varepsilon_t$ where $\theta = {K_T - K_1}/{T - 1}$

K_T is the last calculated k_t while K_1 is the first one. θ is the drift parameter (0.6651) and $T - 1$ is the time difference.

Year	Actual q_x	Calculated q_x
2000	0.071185	0.040894
2001	0.070814	0.04291
2002	0.062899	0.041828
2003	0.050298	0.042046
2004	0.045184	0.040976
2005	0.041878	0.043283
2006	0.032777	0.041975
2007	0.03242	0.040154
2008	0.045277	0.038241
2009	0.033579	0.036198
2010	0.030287	0.037469
2011	0.027292	0.034054
2012	0.025232	0.033892
2013	0.038992	0.033997

Below[2] is an excel extract showing the calculated k_t's for years 2014 through to 2023. They are increasing showing an improving general trend in mortality.

With all the parameters calculated, a projection for a 72-year-old for the next ten years is done. When compared to the mortality rates used by life offices and pension schemes, there is a huge disparity meaning life offices are underestimating mortality. The results suggest the need to develop an efficient model for population dynamics.

Even if a certain mutualisation between mortality and longevity risks obviously exists, it is very difficult to obtain a significant risk reduction between the two, because of their different natures; mortality risk is a short-term risk (1 to 5-year maturity) with a catastrophic component and longevity risk is a long-term risk with maturities ranging from 20 to 80 years (Barrieu, et al., 2012).

The impact of a catastrophe on mortality is really different from the impact on longevity. An abnormally high death rate at a given date has a reduced influence on the longevity trend. Using medical data, one could hope to get better mortality projections by considering

[2] Appendix 1.0

improvements in different mortality causes for each age class. Unfortunately, up to now, this promising approach is far from being possible.

Results show that the gap in the net present value of annuity payments between taking into account mortality and life expectancy improvements or not, is inversely related with the age of pension fund members. In this particular case, the calculations reported assume that life expectancy at birth and at age 65 increases.

As a result of taking into account these improvements in mortality and life expectancy, benefit payments to a 25 year old member will increase by almost ¼ with respect to the case when no account for improvements is taken. Therefore, pension funds with an older age-membership structure will experience an impact from longevity risk on their liabilities and may have less room to maneuver and correct the changes in longevity risk.

Conclusion

The analysis of mortality risk is "at the core of the reinsurance skill set." We are used to carrying out these analyses with due care." (Gavin Jones, July 2013 issue of Reinsurance News). One of the things reinsurers always look at is the difference between lives-based and amounts-based mortality experience.

An unusually large lives-to-amounts differential is an indicator of heterogeneity within the portfolio, which calls for a detailed portfolio-specific mortality analysis, because the portfolio likely consists of different socio-economic groups with varying mortality experience. In life reinsurance pricing, bespoke mortality assumptions are created for each transaction and mortality is differentiated very accurately by risk class.

Groups with different mortality must be shown separately, even if the portfolio's experience data does not provide sufficient information to differentiate the mortality assumptions. For instance, it may be possible that the experience data does not deliver statistically credible results for pension-amount differentials. In such instances, it is necessary to rate the mortality of the projected portfolio based on additional external data. An alternative is using an average mortality rate which may give reasonable results for the initial years.

However, over time the subgroups with higher mortality become less important as their weight naturally decreases faster than the weight of subgroups with lighter mortality. Thus, the required reinsurance premium would be underestimated.

Recommendation

Hedging longevity risk

Longevity risk hedging is most concerned with the financial impact of adverse developments in the long-term trend of survival probabilities. Over the last few years, hedging instruments emerged to help financial institutions e.g. pension funds to protect themselves against longevity risk. They include:

Buy-ins and buy-outs

These involve pension funds buying bulk annuities from insurance companies that then pay out the pensions, therefore, taking over the longevity and investment risk.

In a buy-in, the insurance company makes periodic payments like those pension plan provides to its members, at a fee. This is a way of reducing the risk associated with longevity. However, the sponsor remains responsible for the pension's benefits.

Longevity swaps

In a longevity swap, a pension scheme transfers the risk of paying for its pensioners living longer than expected to counterparty, usually an investment bank or insurance company, in return for an agreed stream of payments from the scheme. Longevity swap reinsurance should be a standard pricing exercise for professional life reinsurers. However, there are several pitfalls along the way which we have highlighted in this article; □ Portfolio-specific mortality is crucial. □ Different risk classes must be projected separately, to avoid under-pricing. □ Mortality improvement trends come with considerable model uncertainty, longevity basis risk and lack of robustness, all of which must be priced for in the risk margin.

Life reinsurers are well suited to take on longevity risk, because they have the required skill set, and because they are likely to require the least amount of additional capital to cover longevity risk. Nevertheless, their capacity to take on this risk is finite. Possibly, longevity swaps will be a tool with which the insurance market will be able to transfer this risk into the capital markets.

However, one of the thresholds to overcome before we will be able to accomplish that is to better understand and quantify longevity basis risk.

Having shown its main characteristics and the state of the art in its understanding and modelling, longevity risk appears to be very complex, due to its specificities compared with other insurance risks - the trend sensitivity, the geographical variability and the associated long-term maturities - and its potential correlations with other sources of risk, financial and non-financial. The insurance industry and especially the life insurance sector are recently adopting new regulations not only to allow for a more accurate risk assessment but also to impose more effective solvency risk management rules.

Those regulations and the increasing convergence between insurance and capital markets have opened the way for alternative risk management solutions and innovative risk transfers. To support the emergence of this new market, not only any asymmetry of information between the various agents involved in potential transactions must be reduced but it is also necessary to develop specific pricing methods and partial hedging methodologies, well-suited to the features of longevity risk and to the immature and illiquid state of the current market.

Additional challenges appear naturally, especially related to the modelling of long term interest rate due to the long maturities of the potential transactions. Longevity risk is far from being a concern for the insurance industry alone. It is indeed at the core of an open discussion for politicians, economists and strategists, who must determine the" effective" retirement age and the" effective" pension scheme as the future generations will almost surely face one of the greatest challenges with an increasing life expectancy.

The potential impacts of longevity risk at various levels of the global economy and society make a better management of this risk one of the key challenges of the coming decade. Interactions with other sources of risk, such as dependence, economy, or ecology, have not been investigated in-depth in this project. But it would be certainly very interesting to study at a macro-level the impacts of longevity on the whole economy and the environment.

The hedging of longevity risk has traditionally been managed by life insurance companies and reinsurance companies. Individuals can hedge their individual longevity risk through various products such as life annuity products, reverse mortgages and indexed annuity products with longevity guarantees. Life insurance companies have traditionally used actuarial solvency and risk management methods based on capital and participating policy design to manage longevity risk.

Longevity risk products are usually exposed also to interest rate, credit, and equity market risks. Except for longevity these risks have well developed markets for hedging and risk management. Rapid increases in longevity have caught pension funds and insurance companies unawares and traditional risk management techniques have been found wanting.

The risk is significant and increasing in many countries internationally. Financial markets are developing an appetite for alternative risk including insurance linked securities. There are benefits for them to hold this risk including the risk premiums to be earned, their risk management expertise, economies of scale and potential cost savings. Many factors suggest that the longevity market will develop further. The recent development of mortality indices for use in financial product design, success of mortality securitizations and activity of the research program in the area has laid a foundation for the wholesale market to develop to meet the needs of an increasing retail market.

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Appendices

	2000	ln 2000	$Z_{x},0$	2001	ln 2001	$Z_{x},1$	2002	ln 2002	$Z_{x},2$	2003	ln 2003
0	0.010671	-4.54023	-0.31682	0.010945	-4.51487	-0.34217	0.009985	-4.60667	-0.25037	0.009687	-4.63697
1-4	0.000587	-7.44049	-0.26735	0.000679	-7.29489	-0.41295	0.000842	-7.07973	-0.62811	0.000782	-7.15366
5-9	0.00038	-7.87534	-0.29609	0.000362	-7.92387	-0.24756	0.000438	-7.73329	-0.43814	0.000303	-8.10178
10-14	0.000327	-8.02555	-0.26987	0.000316	-8.05977	-0.23565	0.000225	-8.39941	0.103994	0.000246	-8.31018
15-19	0.000887	-7.02767	-0.30897	0.00088	-7.03559	-0.30104	0.00081	-7.11848	-0.21816	0.000744	-7.20347
20-24	0.001645	-6.41001	-0.43645	0.001443	-6.54103	-0.30543	0.00155	-6.4695	-0.37696	0.001294	-6.65002
25-29	0.001944	-6.24301	-0.24529	0.001873	-6.28021	-0.20808	0.00174	-6.35387	-0.13443	0.001692	-6.38184
30-34	0.002483	-5.99829	-0.12532	0.002697	-5.91562	-0.20799	0.002668	-5.92643	-0.19718	0.002096	-6.16772
35-39	0.003673	-5.60675	-0.14236	0.003894	-5.54832	-0.20079	0.003728	-5.59188	-0.15722	0.003318	-5.70839
40-44	0.005483	-5.2061	-0.15538	0.005582	-5.18821	-0.17328	0.005161	-5.26662	-0.09486	0.005081	-5.28225
45-49	0.007675	-4.86979	-0.08498	0.008278	-4.79415	-0.16061	0.007968	-4.83232	-0.12245	0.007598	-4.87987
50-54	0.011558	-4.46038	-0.1152	0.01174	-4.44475	-0.13082	0.011378	-4.47607	-0.0995	0.011235	-4.48872
55-59	0.01572	-4.15282	-0.07405	0.016274	-4.11819	-0.10869	0.015997	-4.13535	-0.09152	0.015652	-4.15716
60-64	0.020992	-3.86361	-0.03256	0.021023	-3.86214	-0.03404	0.021474	-3.84091	-0.05526	0.02109	-3.85896
65-69	0.030347	-3.49506	-0.09684	0.029655	-3.51812	-0.07377	0.029502	-3.5233	-0.0686	0.028668	-3.55197
70-74	0.040894	-3.19677	-0.04764	0.04291	-3.14865	-0.09576	0.041828	-3.17419	-0.07022	0.042046	-3.16899
75-79	0.060855	-2.79926	-0.02943	0.062581	-2.77129	-0.05739	0.061454	-2.78947	-0.03922	0.062514	-2.77236
80-84	0.103801	-2.26528	-0.07584	0.105874	-2.24551	-0.09562	0.104606	-2.25755	-0.08357	0.104218	-2.26127
85-89	0.168142	-1.78295	-0.06358	0.174446	-1.74614	-0.10038	0.171557	-1.76284	-0.08368	0.178058	-1.72565
90-94	0.264023	-1.33172	-0.04937	0.269361	-1.3117	-0.06938	0.263103	-1.33521	-0.04588	0.263095	-1.33524
95-99	0.364475	-1.0093	0.034461	0.384223	-0.95653	-0.0183	0.360675	-1.01978	0.044942	0.407567	-0.89755
100-104	0.543421	-0.60987	-0.12114	0.574828	-0.55368	-0.17733	0.468696	-0.7578	0.026788	0.519217	-0.65543
B_x	.		-3.32006	.		-3.75705	.		-3.07961	.	

	Z_x,3	2004	ln 2004	Z_x,4	2005	ln 2005	Z_x,5	2006	ln 2006	Z_x,6	2007
0	-0.22007	0.009249	-4.68324	-0.17381	0.008007	-4.82744	-0.02961	0.007702	-4.86628	0.00923	0.008807
1-4	-0.55418	0.000519	-7.56361	-0.14423	0.000494	-7.61298	-0.09486	0.000438	-7.73329	0.025455	0.000396
5-9	-0.06965	0.000387	-7.85709	-0.31434	0.000292	-8.13876	-0.03267	0.000342	-7.9807	-0.19073	0.000277
10-14	0.014763	0.000223	-8.40834	0.112923	0.000341	-7.98363	-0.31179	0.000238	-8.34324	0.047824	0.000336
15-19	-0.13316	0.000739	-7.21021	-0.12642	0.000671	-7.30674	-0.02989	0.00064	-7.35404	0.017411	0.000714
20-24	-0.19645	0.001206	-6.72045	-0.12602	0.001254	-6.68142	-0.16505	0.001083	-6.82802	-0.01844	0.00089
25-29	-0.10645	0.001476	-6.51842	0.030123	0.001683	-6.38718	-0.10112	0.001586	-6.44654	-0.04176	0.001845
30-34	0.044118	0.002436	-6.0174	-0.10621	0.002449	-6.01208	-0.11153	0.002455	-6.00963	-0.11398	0.0023
35-39	-0.04071	0.003186	-5.74899	-0.00012	0.003723	-5.59323	-0.15588	0.003818	-5.56803	-0.18108	0.003565
40-44	-0.07924	0.005023	-5.29373	-0.06776	0.005484	-5.20592	-0.15556	0.005353	-5.2301	-0.13139	0.005289
45-49	-0.0749	0.007376	-4.90952	-0.04524	0.007327	-4.91619	-0.03858	0.008348	-4.78573	-0.16903	0.007791
50-54	-0.08685	0.011123	-4.49874	-0.07683	0.011602	-4.45658	-0.119	0.012036	-4.41985	-0.15572	0.011568
55-59	-0.06972	0.015046	-4.19664	-0.03023	0.01558	-4.16177	-0.06511	0.016831	-4.08453	-0.14234	0.015563
60-64	-0.03722	0.02101	-3.86276	-0.03342	0.022178	-3.80865	-0.08752	0.022934	-3.77513	-0.12104	0.021995
65-69	-0.03992	0.02853	-3.5568	-0.0351	0.028209	-3.56811	-0.02378	0.028175	-3.56932	-0.02258	0.027782
70-74	-0.07542	0.040976	-3.19477	-0.04964	0.043283	-3.14	-0.10442	0.041975	-3.17068	-0.07373	0.040154
75-79	-0.05632	0.062425	-2.77379	-0.0549	0.061717	-2.7852	-0.04349	0.062281	-2.7761	-0.05259	0.063186
80-84	-0.07985	0.100947	-2.29316	-0.04796	0.102899	-2.27401	-0.06712	0.098545	-2.31724	-0.02388	0.099816
85-89	-0.12088	0.169434	-1.77529	-0.07123	0.167089	-1.78923	-0.05729	0.160942	-1.82671	-0.01981	0.169337
90-94	-0.04585	0.269815	-1.31002	-0.07107	0.261469	-1.34144	-0.03965	0.262182	-1.33872	-0.04237	0.273535
95-99	-0.07729	0.392745	-0.93459	-0.04024	0.38764	-0.94768	-0.02716	0.406317	-0.90062	-0.07421	0.42307
100-104	-0.07558	0.401451	-0.91267	0.181657	0.467908	-0.75948	0.028471	0.541935	-0.61261	-0.1184	0.513524
B_x	-2.18084	.		-1.29007	.		-1.8326	.		-1.59317	.

	ln 2007	Z_x,7	2008	ln 2008	Z_x,8	2009	ln 2009	Z_x,9	2010	ln 2010	Z_x,10
0	-4.73221	-0.12484	0.006763	-4.99629	0.139244	0.00735	-4.91305	0.05601	0.005378	-5.22544	0.368394
1-4	-7.8341	0.126259	0.000343	-7.97778	0.269943	0.000407	-7.8067	0.09886	0.000364	-7.91836	0.21052
5-9	-8.19149	0.020063	0.000199	-8.52221	0.350775	0.000289	-8.14908	-0.02235	0.000253	-8.28212	0.110691
10-14	-7.9984	-0.29702	0.000284	-8.16654	-0.12888	0.000286	-8.15952	-0.1359	0.000391	-7.8468	-0.44861
15-19	-7.24463	-0.092	0.000707	-7.25448	-0.08215	0.00051	-7.5811	0.244468	0.000688	-7.28172	-0.05491
20-24	-7.02429	0.177825	0.00112	-6.79443	-0.05204	0.000916	-6.99549	0.14903	0.000905	-7.00758	0.161112
25-29	-6.29528	-0.19302	0.001615	-6.42842	-0.05988	0.001573	-6.45477	-0.03353	0.001198	-6.7271	0.238805
30-34	-6.07485	-0.04876	0.00213	-6.15163	0.028027	0.002012	-6.20863	0.085019	0.001816	-6.31112	0.187512
35-39	-5.63659	-0.11251	0.002989	-5.81282	0.063711	0.002789	-5.88207	0.132967	0.002963	-5.82155	0.072448
40-44	-5.24213	-0.11936	0.004388	-5.42888	0.067396	0.003981	-5.52622	0.164737	0.004471	-5.41014	0.048658
45-49	-4.85479	-0.09998	0.007012	-4.96013	0.005364	0.006167	-5.08854	0.133775	0.006852	-4.98321	0.028446
50-54	-4.45951	-0.11606	0.009806	-4.62476	0.049188	0.00892	-4.71946	0.143886	0.009691	-4.63656	0.060985
55-59	-4.16286	-0.06402	0.014354	-4.24373	0.016851	0.013889	-4.27666	0.049782	0.014052	-4.26499	0.038115
60-64	-3.81694	-0.07924	0.020663	-3.87941	-0.01677	0.019453	-3.93975	0.043578	0.01954	-3.93529	0.039116
65-69	-3.58337	-0.00853	0.026601	-3.62681	0.03491	0.025733	-3.65998	0.068084	0.028621	-3.55361	-0.03828
70-74	-3.21503	-0.02938	0.038241	-3.26385	0.019434	0.036198	-3.31875	0.074338	0.037469	-3.28424	0.039828
75-79	-2.76167	-0.06701	0.059512	-2.82158	-0.00711	0.057975	-2.84774	0.019056	0.059242	-2.82612	-0.00256
80-84	-2.30443	-0.0367	0.093838	-2.36619	0.025062	0.091061	-2.39623	0.055102	0.08807	-2.42962	0.088499
85-89	-1.77586	-0.07066	0.15854	-1.84175	-0.00477	0.147252	-1.91561	0.069087	0.130001	-2.04021	0.19369
90-94	-1.29633	-0.08476	0.244218	-1.40969	0.028607	0.252926	-1.37466	-0.00643	0.209727	-1.56195	0.180861
95-99	-0.86022	-0.11462	0.396943	-0.92396	-0.05087	0.392211	-0.93596	-0.03888	0.29862	-1.20858	0.233747
100-104	-0.66646	-0.06455	0.521264	-0.6515	-0.07951	0.520608	-0.65276	-0.07825	0.309781	-1.17189	0.440877
B_x		-1.49887	.		0.616529	.		1.272447	.		2.197936

	2011	ln 2011	Z_x,11	2012	ln 2012	Z_x,12	2013	ln 2013	Z_x,13
0	0.006555	-5.02753	0.170482	0.006524	-5.03227	0.175223	0.004534	-5.39615	0.539106
1-4	0.000327	-8.02555	0.317713	0.000209	-8.47318	0.765339	0.000337	-7.99543	0.287591
5-9	0.000251	-8.29006	0.118627	0.000175	-8.65072	0.479294	0.000166	-8.70352	0.532092
10-14	0.000156	-8.76565	0.470239	0.000101	-9.20039	0.904974	0.00021	-8.4684	0.172987
15-19	0.000498	-7.60491	0.268279	0.000528	-7.54641	0.209783	0.000355	-7.94339	0.606761
20-24	0.00077	-7.16912	0.322656	0.000751	-7.1941	0.347641	0.000633	-7.36504	0.518576
25-29	0.001218	-6.71055	0.222249	0.001172	-6.74904	0.260747	0.001049	-6.85992	0.371622
30-34	0.001855	-6.28987	0.166264	0.00167	-6.39493	0.271325	0.001926	-6.25231	0.128703
35-39	0.002466	-6.00516	0.256053	0.002483	-5.99829	0.249183	0.002566	-5.96541	0.216302
40-44	0.003602	-5.62627	0.264781	0.003721	-5.59376	0.232278	0.003847	-5.56046	0.198976
45-49	0.005837	-5.14354	0.18877	0.005537	-5.1963	0.241534	0.005784	-5.15266	0.197892
50-54	0.008251	-4.79742	0.221848	0.008231	-4.79985	0.224275	0.008435	-4.77537	0.199793
55-59	0.012825	-4.35636	0.129483	0.011776	-4.44169	0.214816	0.011992	-4.42352	0.19664
60-64	0.017648	-4.03713	0.140957	0.018141	-4.00958	0.113405	0.017315	-4.05618	0.160006
65-69	0.026278	-3.63902	0.047126	0.024722	-3.70006	0.108165	0.02373	-3.74102	0.149118
70-74	0.034054	-3.37981	0.135394	0.033892	-3.38458	0.140163	0.033997	-3.38148	0.13707
75-79	0.053311	-2.93161	0.102925	0.051767	-2.961	0.132315	0.050569	-2.98442	0.155729
80-84	0.08494	-2.46581	0.124686	0.088896	-2.42029	0.079164	0.083814	-2.47916	0.138031
85-89	0.137345	-1.98526	0.138736	0.146256	-1.9224	0.075873	0.140657	-1.96143	0.114908
90-94	0.228328	-1.47697	0.095885	0.235423	-1.44637	0.065284	0.23103	-1.46521	0.08412
95-99	0.357316	-1.02913	0.054298	0.368439	-0.99848	0.023644	0.358681	-1.02532	0.050486
100-104	0.405949	-0.90153	0.170515	0.572499	-0.55774	-0.17327	0.462668	-0.77075	0.039733
B_x			4.127968			5.141157			5.196242

Appendix 1.1