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Empirical performance of alternative risk measures in portfolio selection - the case of South African stock market.

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Empirical performance of alternative risk measures in portfolio selection

The case of South African stock market

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Declaration

I declare that this work has not been previously submitted and approved for the award of a degree by this or any other University. To the best of my knowledge and belief, the dissertation contains no material previously published or written by any other person except where due reference is made in the dissertation itself.

Richard Macharia Mwangi

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June 27^{th}, 2021

Approval

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Chapter 1

Introduction

1.1 Background to the study

Portfolio selection is the process of apportioning capital to a finite number of assets given the wider set of all investment options. The decision of best combination of assets to invest in is the subject of debate among practitioners and researchers alike.

Individuals face a multitude of constraints when making allocation decisions thus their patterns of investing are wildly different. However, economists have studied asset price patterns for long enough to be able to pick out aggregate patterns and develop a theory of decision making: Utility Theory.

Under utility theory, all the benefits that an individual derive from their asset allocation decision is summarised into one value: utility. Rational individuals are able to rank different asset portfolios based on the "satisfaction" they can derive from them. Therefore, all decisions are made on the basis of maximizing utility Back (2017). In the context of a stock market, it is reasonable to assume that this satisfaction can be approximated by a function of risk and return.

Modern portfolio theory, whose origin links to Markowitz (1952), extends this thinking and concludes that investors measure their utility in portfolios using expected Return(mean) and variance of asset returns. Given these two measures, Markowitz (2010) was able to create a portfolio selection method that maximizes the return for a given level of risk. The method is both a hypothesis about how investors make their decisions (positive economics) and a recommendation about what investors should do (normative economics).

As acceptance of the Markowitz method grew, researchers focused their efforts on creating a method of measuring portfolio performance that would work in the context of the Markowitz theory. This led to the develpoment of the Sharpe ratio by Sharpe (1966) then called reward-to-variability. It is a simple ratio of expected return to volatility of the portfolio and since then, it has become one of the most popular methods of ranking portfolio performance in practical application. Maximizing the sharpe ratio is the common form of Mean Variance Optimization where a risk free asset exists.

As more researchers put the markowitz theory to the test, it became clear that it has severe limitations. For instance, estimation errors in both the expected returns and the co-variance matrix hamper the construction of reliable portfolios within the Markowitz framework which often leads to poor out of sample performance Biglova, Ortobelli, Rachev, and Stoyanov (2004).

1.2 Problem Statement

To this day, the mean variance optimization method remains the most commonly taught method to finance practitioners and consequently, it the most commonly used method in industry. This is so despite its shortcomings of: Poor out of sample performance, Undiversified portfolios, difficulty in estimating a expected return and a variance for assets, Mean Variance preferences are not well approximated by quadratic utility if returns fall outside range of (-30%, 40%) according to Markowitz (2010)

Multiple researchers have attempted to address these shortcomings by creating new methods of portfolio selection that incorporate subjective investor views(e.g Black-Litterman method), make use of alternative risk reward measures(e.g. Rachev ratios), use higher moments for measuring risk or focus on maximizing returns without considering return(e.g. online portfolio methods). Despite all these advancements there is no overall conclusion on the best choice of model for selecting portfolios.

Additionally, the literature on portfolio selection in african markets remains scarce. Of the multitude of methods in existence for portfolio selection, only a few researchers have examined the performance of those methods in african emerging or frontier markets. This thesis comes in to fill in that gap by comparing the performance of alternative risk measures in selecting portfolios from the South African stock market.

1.3 Objectives

The main objective of this study is to determine the best risk measure to use for portfolio selection in the South African market. The specific risk measures to be compared are:partial moments, conditional moments, mean drawdown, quantiles, expected shortfall and variance.

The risk measures are to be evaluated on sharpe ratio, final wealth, minimum return, average return and median return.

Additionally, all the above methods will be compared to the naive equally weighted portfolio.

1.4 Significance of the study

This thesis is useful to practitioners who are constantly facing asset allocation decision in emerging and frontier markets in Africa. The study offers an analysis of possible returns that would have been earned if asset allocation decisions were made based on alternative risk measures. The analysis focuses more on the South African market as it is one of the top 3 economies in Africa. Moreover,south Africa has more companies listed [1] than Nigeria and Egypt which are ranked in first and second position respectively.

[1] 367 as time of writing

Chapter 2

Literature review

2.1 Background

Mean Variance analysis was proposed by Markowitz (1952) as a portfolio construction framework that bases its allocation decision on the minimum level of risk acceptable for a specified expected return. It's basis is that investors will favor a portfolio with a lower risk level over a higher risk level for the same level of return. Additionally, Markowitz theory introduces the concept of co-variance of assets in a portfolio and suggests that investors look at how an individual security impacts the risk and return profile of an entire portfolio as opposed to assessing it separately. Mean Variance method assumes that investors have reasonable beliefs about future expected returns and covariance.

Although the mean variance method is fully compatible with normally distributed returns [1], it will lead to incorrect investment decisions when returns present kurtosis or skewness.

2.1.1 On the distribution of asset returns

From Cont (2001) stylized facts of stock returns, we can see that return distribution is governed by more that expectation and variance. For instance, stationarity of returns is very dependent on the time scale chosen and even then different stationary tests show different results depending on the time interval chosen. This is in line with the observation of seasonality in returns. Therefore, sample mean and variance of various stock returns do not necessarily represent the mean and variance of the population. This in turn affects the soundness of portfolios selected by the mean variance method.

2.1.2 Alternative risk-reward ratios

It is possible to construct a return measure that is a function of asset characteristics other than expected return as shown by Scherer (2007). These characteristics include: price to book, size, sentiment, beta etc.

Several alternatives to the Sharpe ratio for optimal portfolio selection have been proposed such as the minimax ratio, the stable ratio, the mean absolute deviation ratio, the Farinelli-Tibiletti ratio, and the Sortino-Satchell ratio.

Sortino-Satchell ratio is a modification of the sortino ratio which penalizes portfolio excess return by downside deviation Pedersen and Satchell (2002). The measure for downside deivation used is semi-variance, which is the second partial moment. The

[1] or, in general, with elliptical returns

ratio is defined as:

$$\rho(r_i, r_f) = \frac{r_i - r_f}{\sqrt{P_2^-}} \tag{2.1}$$

$$P_2^- = \int_{-\infty}^{0} (r-\mu)^2 f(r) dr \tag{2.2}$$

Value at Risk(VaR) provides a single value for the possible loss that could occur with a given probability α within a given horizon.Conditional Value at Risk (CVAR) Measure the expected value of loss given that VaR has been exceeded. It is also referred to as the expected shortfall.

This Rachev ratio compares the expected tail return to the expected tail losses. The two tails do not necessarily have to be from the same confidence level. The Rachev generalized ratio adds a parameter to characterize an investor's risk aversion Biglova et al. (2004). Using daily data for a period of 3 years, the authors were able to show that the Rachev ratios were able to generate superior end of period wealth when used to select portfolios compared to sharpe ratio and all other previously mentioned ratios. The Rachev ratio is given as:

$$\rho(r, r_f) = \frac{ETL_(\alpha)(r_f - r)}{ETL_(\beta)(r - r_f)} \tag{2.3}$$

where ETL is the expected tail loss and α, $\beta \in (0, 1)$ are the confidence levels

Tibiletti and Farinelli (2002) created their own ratio that heads a step beyond the classical approach of equally weighting upside and downside risk in order to handle with asymmetrical return distributions and enhance asymmetrical preferences. Their ratios is not only about distinguishing variation above and below a benchmark but also distinguishing small variations form larger ones. The ranking ratio created s proved to be compatible with the expected utility approach.

Fastrich Fastrich and Winker (2012) propose a modification to the MVO model that eliminates oversimplification of real world scenarios

2.2 Portfolio selection in African markets

Masese, Othieno, Njenga, et al. (2017) find that the threshold acceptance model is more consistent than the Mean variance model when tested in the Kenyan market for both weekly and monthly returns. Threshhold acceptance(TA) model tends to choose fewer assets for a portfolio as shown in table 3 and 4 of their paper. Additionally the TA method was able to select a portfolio that is more diversified across industries. The TA selected portfolio overall caries higher risk when measured using sharpe and sortino ratios.This paper concludes that the TA model outperforms the MVO model for portfolio selection but it does not perform well consistently over different time periods.

Petje (2016) constructs optimal portfolios using mean variance method using equity and bond assets from the South African market. The paper uses 16 years worth of data in backetsting and finds that the mean variance method underperforms actively managed mutual funds when compared on return earned. However, this conclusion reverses if the comparison is based on sharpe ratio instead. Notably, the paper assesses mean both with and without a return constraint of beating inflation.

Nkomo, Kabundi, et al. (2013) examine online portfolio selection method based on a price relative derived from the kalman filter. Online portfolio selection methods typically involve applying machine learning methods to sequentially select the best portfolio at each point in time. These methods do not typically involve risk reduction but rather focus solely on increasing expected return. The authors assess the performance of this method in several markets one of which is the Johannesburg Stock Exchange. Using data for 22 years, the authors find that final wealth gained from their method is up to 7800 times the initial investment.

Nyokangi (2016) compared the performance of the single index model and the MVO model using 13 years worth of data from the Nairobi stock exchange. The study concluded that the mean variance model is better for longer investment horizons, whereas the single index model is better when considering investments with short time horizon. It was however suggested that other models could be considered and a different performance measure could be used because of the weaknesses of the Sharpe ratio in accuracy when stocks are skewed.

Oyatoye, Okpokpo, and Adekoya (2010) advocates for more risk sensitive approaches to investment decision making in the Nigerian Markets using the analytic hierarchy process(AHP). AHP provides a rational framework for a needed decision by quantifying its criteria and alternative options, and for relating those elements to the overall goal. The AHP method is based on a questionnaire that looks at 5 levels of factors affecting the investment decision. The study was focused on an asset universe of 12 banks operating in Nigeria. A highlight of their conclusions include: Most investors in Nigerian capital market were passive investors who just buy shares and keep, and not really speculating for capital gains, Financial criterion was found to be the most critical in terms of investing in stocks of commercial banks in Nigeria.

2.3 Coherent risk measures

Fischer (2003) assesses the justification of risk measures to be used in portfolio allocation. He does this by examining the differentiability properties. The paper defines a class of coherent risk measures which depend on the mean and the one-sided higher moments of a risky position. These measures are given by:

$$\rho_{p,a}(X) = -E_Q(X) + a\sigma_p(X) = -E_Q(X) + a|(X - E_Q(X))|_p^-$$

They illustrate how this measure can be used to allocate risk capital given by the VaR can be allocated. This works for both continuous and discrete distributions.

2.4 Comparison of empirical methodologies

Multiple methods have been used to compare portfolio selection criteria. Below is a summary of methodologies in papers reviewed.

Masese et al. (2017) use an in sample performance ranking of threshold accepting method versus Mean variance analysis. That is the performance of the methods is tested on the same data used to generate the optimal portfolios. This could have biased their results to the sample period and does not speak to the out of sample performance of the model. Additionally, the authors compare consistency of the methods by comparing performance using both weekly and monthly returns such that, a more consistent model is expected to select similar stocks despite the return horizon considered.

Gilli and Schumann (2009) compare multiple risk measures constructed from moments, drawdowns, quantiles, Value at risk and more against the Mean Variance method. Their method is based on generating a distribution of portfolio returns by simulating an investors who construct their portfolio every three months using previous one year data. Each investor selects a portfolio by minimizing on of the aforementioned risk measures using threshold accepting optimization. The portfolio returns after the three months for all investors after every 3 months are all stored and used to generate box plots.

The determination of the best methods is then based on the median of their out of sample returns. This provides a more robust approach of comparison as it tests a portfolio independently of the data that generated it.

Dione, Diagne, Dione, and Gningue (2015) compare the performance of the Mean variance model with an altered form that introduces a constraint on the price earnings ratio. This ratio is a proxy for the number of years required to recover an investment. The authors prove that their model is at least as good as the mean variance model by proving that it converges to the same efficient frontier as MVO but with fewer assets. The proof is based on numerical experiments as well as mathematical principals. However, this method of comparison is not as straightforward to understand for practitioners in the investment industry.

Ortobelli, Biglova, Stoyanov, Rachev, and Fabozzi (2005) compare different reward measures used in portfolio construction. Their method involves choosing a set of utility functions, together with a set of risk aversion coefficients. Separately they derive the efficient frontier and market portfolio for each of the selected reward measures. They then set out to calculate maximum utility for investors who apportion their wealth between the market portfolio and a risk free asset. The maximization happens across all chosen utility functions and risk aversion coefficients.

Using this large number of simulations the authors were able to approximate expected utility and determine which reward measure offers the highest utility. A similar method was used by Harvey, Liechty, Liechty, and Müller (2010) when comparing bayesian portfolio selection models to non bayesian models.

2.5 Summary

The literature reviewed highlights the alternative risk measures proposed by various authors and presents their performance results. However, not enough literature exists about these methods to make clear conclusions on the best risk measure to be used in african markets. This thesis serves to fill in that gap by comparing performance of alternative risk measures in selecting portfolios from the South African stock market. Specifically, this thesis will test performance of conditional moments, partial moments mean draw down and variance.

Additionally, the literature is abundant with methods of comparing portfolio selection criteria. This thesis opts for a variant of the Gilli and Schumann (2009) method. That is simulating investors who make portfolio decisions on regular intervals and asses their final wealth, portfolio Sharpe ratio and draw down among other return characteristics. The simulation method is chosen because its more relatable to practitioners: money market funds, pension funds to name a few. These attract investors by show casing their historical performance as opposed to their utility calculations or in sample model performance.

Chapter 3

Methodology

3.1 Data and Study Design

The proposed research will compare performance of portfolios constructed by minimising risk measures of moments, quantiles and drawdowns. Additionally, those portfolios will be compared against mean variance optimal portfolios and portfolios of equally weighted assets. The comparison will be done by examining statistical properties returns an investor would have earned had he been investing in the optimal portfolios every five day period for five years. This study uses Daily price series from Johannesburg Stock Exchange(JSE) spanning 6 years from September 2013 up to September 2019.

3.2 Definition of risk measures

3.2.1 Moments

Sample moments centered on the mean are given as:

$$M_\lambda(r) = \frac{1}{n_s - 1} \sum_{s=1}^{n_s} (r_s - \mu)^\lambda$$

Sample partial moments around a desired return are given as:

$$P^+(r_d) = \frac{1}{n_s} \sum_{r \geq r_d} (r - r_d)^\lambda \tag{3.1}$$

$$P-(r_d) = \frac{1}{n_s} \sum_{r \leq r_d} (r_d - r)^\lambda \tag{3.2}$$

The semi-variance is a common partial moment given by $P_2^- 1(M(r))$ in risk capital allocation.

Conditional moments are given by:

$$C^+(r_d) = \frac{1}{\#(r > r_d)} \sum_{r \geq r_d} (r - r_d)^\lambda \tag{3.3}$$

$$C^-(r_d) = \frac{1}{\#(r < r_d)} \sum_{r \leq r_d} (r_d - r)^\lambda \tag{3.4}$$

where the $\#(.)$ symbol is represents the count of returns meeting the specified condition. In simple terms, the conditional moment measures the magnitude of returns around r_d, while the partial moment also takes into account the probability of such returns. This is so because of the denominator used in the sum. Expected shortfall is a special case of conditional moment where the r_d is a specified quartile. All moments used in this thesis are lower moments.

3.2.2 Quantiles

The Q^{th} quantile is given by:

$$Q_q = CDF^{-}1(q) = min\{r|CDF(r) \geq q\} \tag{3.5}$$

here q is the proportion of observations smaller than Q_q. Q_0 and Q_100 represent the sample minimum and maximum respectively. Value at Risk measure is closely related to this as it represents the maximum loss expected, beyond which there is a low probability of occurring.

3.2.3 Drawdowns

The Drawdown of a series of prices is given by:

$$D_t = p_t^{max} - p_t \tag{3.6}$$

where p_t^max is the maximum up to the most recent time point considered. When the drawdown is observed over time, we can create various functions of the drawdown with which to measure risk. These include the maximum drawdown, mean and standard deviation.

3.3 Portfolio construction methods

It is proposed to construct optimal portfolios using the building blocks of partial moments, conditional moments, and drawdown. As such, this thesis tests 10 risk measures which are lower partial moments of order $2(P(\lambda = 2))$, second order lower conditional moment($C(\lambda = 2)$), third order lower conditional moment($C(\lambda = 3)$), 5% quartile(Q_5), 10% quartile(Q_{10}) and 20% quartile(Q_{20}), mean drawdown, 5% Expected Shortfall($ES(q = 5)$), 10% Expected Shortfall($ES(q = 10)$) and 20% Expected Shortfall($ES(q = 20)$)

The optimization problem used in all portfolios selected is as below:

$$\begin{aligned} Min \;\; & \Phi(w) \\ s.t \;\; & w'R \geq \bar{R} \\ & w'\mathbf{1} \geq 0.7 \\ \mathbf{0}\% \leq \; & w \leq \mathbf{15}\% \end{aligned}$$

where $\Phi(w)$ represents the risk measure.

The constraints above are similar to what previous researchers have used thus making findings from this thesis more comparable.

3.3.1 Algorithm for comparison

- Construct optimal portfolios(using all measures) every five day period using preceding 1 year data
- Construct weekly benchmark portfolios using Mean variance and equally weighted portfolio. The mean variance portfolios are also based on previous 1 year data.

- Track weekly performance of constructed portfolios
- Compare performance using box plot of weekly returns of constructed portfolios. Better performing models should have high medians and low dispersion from mean.
- Compare additional return characteristics: sharpe ratio, minimum return, mean, variance, maximum return and final wealth.

Weekly investment horizon is chosen because it yields more observations than monthly or 3 month horizons when applied to a 6 year dataset. Additionally, weekly horizons reduce the high rebalancing costs that occur with trading at daily horizons.

Chapter 4

Research Findings

4.1 Introduction

The research findings sections will be an analysis of the main summary in table 4.1. This table shows all the statistics that were used to judge performance of the different methods: mean, minimium, maximum, percentiles, sharpe ratio and final wealth.

4.2 Distribution comparison

The figure 4.1 above shows the comparison of portfolio median returns where the portfolios are constructed by minimizing the various risk measures. It is evident in this graph that the third order conditional moment($C(lambda = 3)$) has the highest median return at 3.2% weekly. Moreover, Mean Variance returns(MVO) are much lower than those of 20% quantile(q), Expected Shortfall at 20% (ES) and mean drawdown risk measures.

In contrast, the 5% quantile and 10% expected shortfall have below 0 median returns, suggesting that they are not suitable objective functions for portfolio selection.

Unexpectedly, lower semi-variance, a partial moment($P(lambda = 2)$) and one of the oldest MVO alternatives seems to perform worse than MVO and the 10% quantile. Though both of this have median returns that are only slightly above 1%. The performance of the third order moment is in stark constrast with the results of Gilli and Schumann (2009) who advocated for using moments of order 1.5 or lower.

Table 4.1: Summary of weekly returns of portfolios created by minimizing risk measures

	mean	std	min	25%	50%	75%	max	Total Return(%)	Sharpe Ratio
q_5	12.47	57.54	-99.20	-25.57	-0.94	40.99	210.28	-99.99	-1.74
ES(q=10%)	16.91	64.07	-99.78	-26.40	-0.07	46.23	238.73	-99.94	-1.56
Equally Weighted	0.54	3.26	-4.94	-0.44	0.14	1.23	46.51	247.10	75.83
P(lambda_=2)	2.83	21.83	-14.79	-1.29	1.03	4.52	339.59	13453.54	616.36
C(lambda_=2)	2.97	23.45	-12.85	-1.26	1.08	4.46	366.15	15644.92	667.06
q_10	14.00	55.78	-97.01	-23.07	1.10	38.21	204.73	62.57	1.12
MVO	2.42	16.29	-15.41	-1.62	1.23	4.10	246.68	9176.66	563.40
ES(q=5%)	15.19	61.58	-112.39	-27.07	1.60	43.44	220.34	-100.12	-1.63
ES(q=20%)	17.21	64.14	-110.32	-25.38	1.87	44.76	238.24	-105.89	-1.65
DrawDown mean	3.95	16.97	-32.29	-2.46	1.93	7.08	155.44	148692.80	8761.49
q_20	12.44	50.22	-96.46	-20.75	2.13	37.93	209.99	858.22	17.09
C(lambda_=3)	19.60	74.81	-119.84	-26.33	3.17	44.45	613.60	-99.99	-1.34

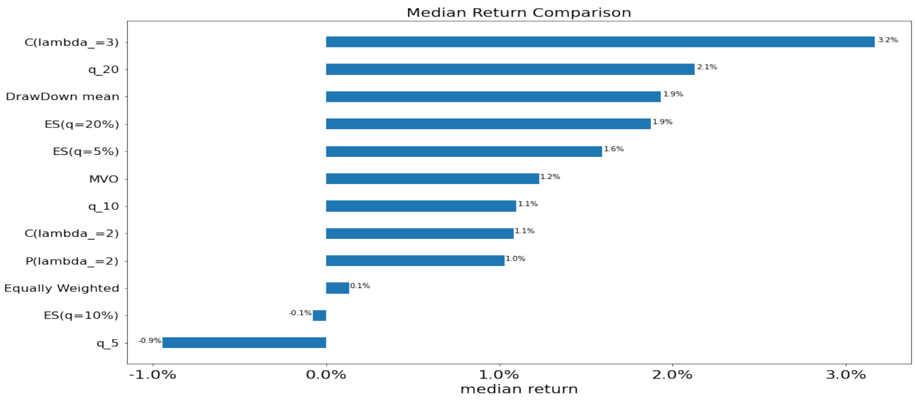

Figure 4.1: Weekly Median Return Comparison across all risk measures

The naive equally weighted portfolio has generated the least positive returns of 0.1% which is only a tenth of the returns of MVO.

In order to get a sense of the dispersion of returns associated with the returns in this portfolios, a box plot is plotted below:

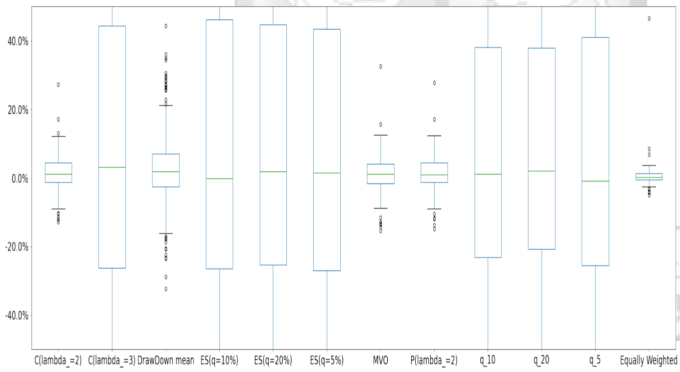

Figure 4.2: Distribution of portfolio returns across all risk measures

From the below box plot, it is evident that the high median return from the third order Conditional Moment($C(lambda = 3)$) comes with a high cost in variability. The returns from this measure have a 25% percentile of -26% and an upper percentile of 44%. (See 4.1 in the appendix for all summary statistics.).

Contrary to previous research, MVO returns from these portfolios are not as sensitive to the data as is commonly described in literature. The MVO has a lower percentile of -1.4% and an upper one of 43%. These puts in the top 4 measures that display low variance, others being 2^{nd} order lower conditional moment($C(lambda = 3)$), mean drawdown and 2^{nd} order lower conditional moment($P(lambda = 2)$) (See 4.1 in the appendix for all summary statistics.).

The equally weighted portfolio has the least variance of portfolio selection methods. This is quite surprising given it does not consider the covariance matrix during portfolio formulation.

4.2.1 Wealth Trajectory

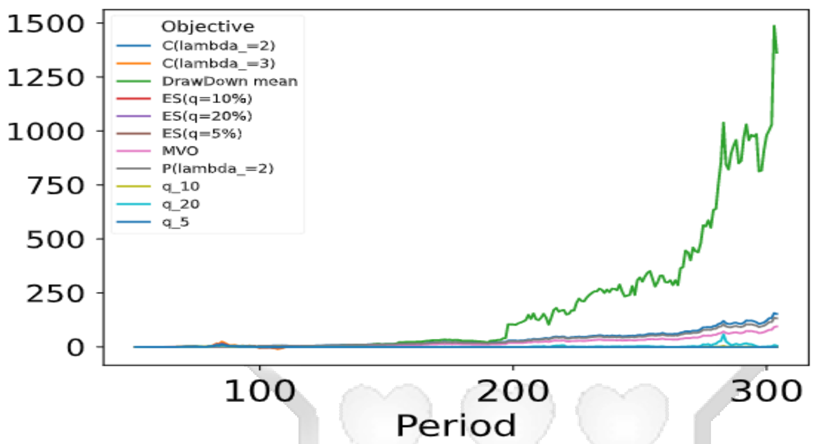

Figure 4.3: Wealth Trajectory over 5 years(300 weeks) of portfolios created by minimizing risk measures with weekly rebalancing

Table 4.2: Final Return of portfolios based on minimizing risk measures

	Total Return(%)
DrawDown mean	148692.80
C(lambda_=2)	15644.92
P(lambda_=2)	13453.54
MVO	9176.66
q_20	858.22
Equally Weighted	247.10
q_10	62.57
ES(q=10%)	-99.94
C(lambda_=3)	-99.99
q_5	-99.99
ES(q=5%)	-100.12
ES(q=20%)	-105.89

Figure 4.3 shows long term trajectory of portfolios formed using the selected risk measures and weekly rebalancing. Mean drawdown is the best performer by far by total wealth. Mean drawdown portfolios return up to 148000% of initial wealth over the 5 year period. Mean Variance method does come in as the fourth best performer with 9100%. See 4.3 for the exact total return numbers.

From 4.3, it is notable that 5% quantile, expected shortfall at 10% percentile have negative total return. This is consistent with the negative weekly median return that they show in 4.1. Coupling this with the observed low sharpe ratios suggests that these measures are not good candidates for portfolio selection.

However, third order conditional moment which had the highest median weekly return has a very poor total return of -99%. This is in part explained by its high standard deviation seen in table 4.1. This suggests that moments beyond the second order may not be good at estimating risk in portfolios, a notably differing recommendation from the work of Gilli and Schumann (2009) who suggested 1.5 as the cut-off order.

Total return of portfolios formed from Expected shortfall(both at 5% and 20%) show negative total returns despite the positive median weekly returns. Conditional Moment of the second order and Second order partial moment do show better final returns than mean variance method at 15600% and 13400% respectively.

4.2.2 All Statistics ranking

In this section, the different risk measures are ranked by their minimum return, maximum return, return quantiles, standard deviation, final wealth and sharpe ratio. These statistics are all ranked in figure 4.4.

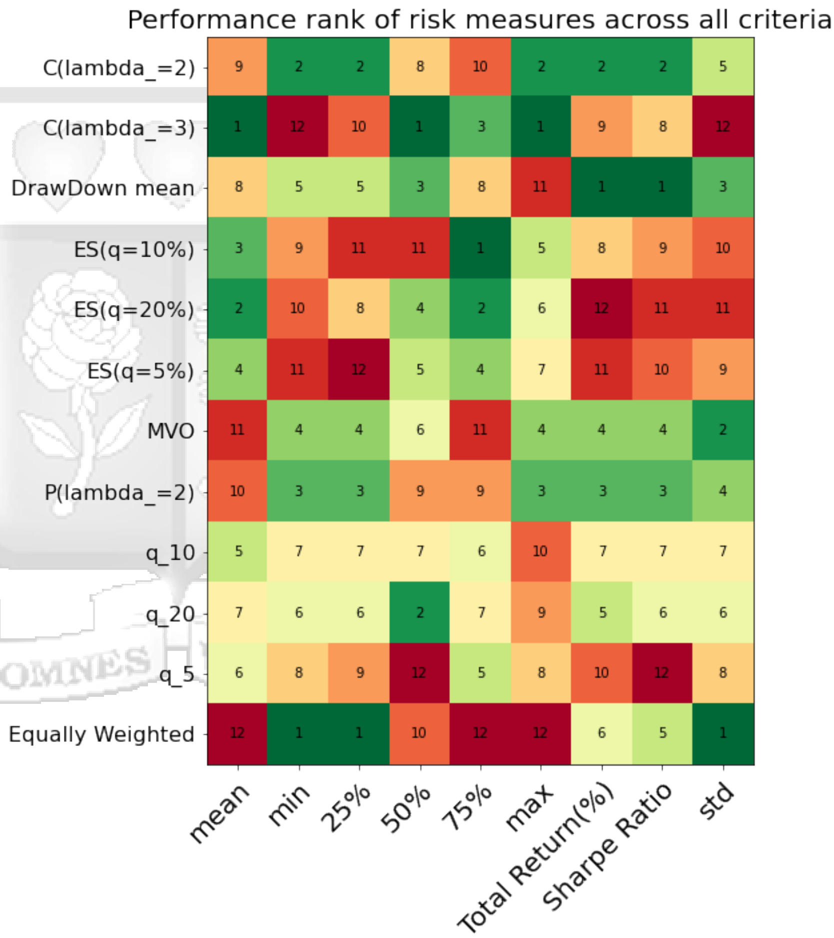

Figure 4.4: Heat map of all portfolio return statistics. Green color and lower ranks represent better performance

The MVO line on the heat map is mostly light green meaning it has average performance across most statistics, but has the second worst mean return and 75% percentile ranked at 11. The MVO is never ranked 1 in any of the criteria.

On the other hand mean drawdown line has only one dark red point, that is the maximum return achieved. From 4.1 this maximum return is at 155% weekly which is the second worst return. This is still much higher than the 46% achieved by an equally weighted portfolio.

The 5% quantile has a mostly red ranking of performance across all statistics. The performance across all criteria is either less than average or exactly average. It is also the worst performer in terms of sharpe ratio. The 10% and 20% quantiles have mostly average ranking across all metrics. The 10% quantile specifically is consistently ranked 7 in 6 of the 9 rankings.

The expected shortfall measures show ambiguous performance metrics in general but they are ranked rather poorly in total return, sharpe ratio and standard deviation. It is notable that they do have high 75% percentile and mean returns with the 10% Expected shortfall in top rank on the former.

The 3^{rd} order conditional moment has the best mean and median returns but performs very poorly when ranked on total return, sharpe ratio and standard deviation. In contrast, the 2^{nd} conditional moment, is an all round good performer save for mean and 75% percentile returns. Of all the moment measures, the 3^{rd} conditional moment, is the only one to have ever received the bottom rank in any criteria, that is, standard deviation and minimum return. The second order partial moment does appear in the top 4 across most measures but does not have good mean or median returns. In addition, this risk measure is not a top performer in any criteria.

The equally weighted portfolio shows mixed signals. It has the best standard deviation and 25% returns but only has average total return and sharpe ratio. It is also shows the worst mean return across all portfolios. The best sharpe ratios across all portfolio come from mean drawdown and second order conditional moment. This puts them as the most likly candidates for further research by practitioners.

Chapter 5

Conclusion

The thesis set out to compare alternative risk measures in their ability to select "good" portfolios in the South African market. The specific risk measures included moments, mean drawdown, expected shortfall and variance. The comparison has been done by simulating investors who have weekly investment horizons and keep trading their portfolios over a 5 year period. A comparison of distribution properties of returns and final wealth across all the selected risk measures has enabled measuring of performance of different risk measures.

This thesis concludes that the mean drawdown, 2^{nd} order partial and conditional moments risk measures perform better than mean variance method for weekly investment horizons based on total return and sharpe ratio. The three measures hold the top 3 position in sharpe ratio and total return. This is in line with observations of Gilli and Schumann (2009) who had examined stocks from the European region. There is a distinction however in the exceptional final wealth trajectory of portfolios created by minimising mean drawdown.

Using the same criteria, this study concludes that expected shortfall and quantiles are at best average performers in selecting portfolios regardless of the confidence level used for evaluating. This comes quite unexpectedly given their prevalent use in banking for risk management.

In addition, no single risk measure is consistently a top performer on all the different criteria. This leaves room for creation of a measure that can combine all the best properties of all the assessed measures.

The significance of these conclusions is that practitioners of the investment industry in South African market should use mean drawdown and moments in the assessment of risk in portfolios. This thesis has also contributed to filling the current gap of literature on how the existing profusion of risk measures and portfolio selection methods perform in african markets. Quite surprisingly, african markets' distinct characteristics of greater political instability, reduced banking regulatory standards, significantly lower liquidity and fewer equity assets to trade seem not to have overpowered drawdown and moments risk measures in selecting portfolios.

The thesis leaves room for future researchers to assess the impact of transaction costs on performance of the risk measures. Practitioners may additionally be interested in measuring the exposure of selected portfolios to existing sectors. The robustness of the results could be examined further by use of longer time interval and possible jack knifing of the data.

Appendix

.1 Code

```
# To add a new cell, type '# %%'
# To add a new markdown cell, type '# %% [markdown]'
# %%
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from matplotlib.ticker import FuncFormatter
from config import length_period, periods_in_year

plt.style.use('seaborn-paper')
plt.rcParams['axes.labelsize'] = 20
plt.rcParams['axes.titlesize'] = 20
plt.rcParams['xtick.labelsize'] = 20
plt.rcParams['ytick.labelsize'] = 16


# %%
returns = pd.read_csv('./data/cleaned_returns.csv').replace([np.inf, -np.inf], np.
returns.loc[:, 'Date'] = pd.to_datetime(returns.loc[:, 'Date'], format='%Y-%m-%d')
returns = returns.set_index(keys=['Date'], drop=True)
weights = pd.read_csv('./results/zero_target_return_optimizations.csv')
.drop(columns=['Unnamed: 0'], errors='ignore')              .fillna(0)


# confirm weights and returns have same number of stocks as columns
assert (len(returns.columns)-1) == (len(weights.columns)-4)


combined = returns.reset_index().rename(mapper={'period_count':'Period'}, axis=1)
.merge(right=weights, on='Period',
                     how='inner', suffixes=('#return', '#weight'))\
               .set_index(keys=[ 'Objective',
                            'Constraints', 'Period', 'Asset Count', 'Date']
               .sort_index(axis=0, level=[0,1,3,4])
assert all(['#' in col for col in combined.columns])
columns = ([col.split('#')[1], col.split('#')[0]] for col in combined.columns)
combined.columns = pd.MultiIndex.from_tuples(columns, names=('Type', 'Stock'))
combined.head(10)


# %%
# creating portfolio return from weights and stock returns
weights = combined.loc[:, ('weight', slice(None))].rename(columns={'weight':'opera

massaged_returns = combined.loc[:, ('return', slice(None))]
.rename(columns={'return':'operand'}, level=0)
performance = (weights*massaged_returns)
# remove upper column level
```

```
performance.columns = performance.columns.get_level_values(1)
# simple returns are summable across assets
performance = performance.sum(axis=1).to_frame(name='Portfolio Performance') + 1
performance = performance.groupby(level=[0, 1, 2, 3]).prod()
# annualise returns
performance = (performance-1) * 100
performance = performance.unstack(level=[0,1])
performance.columns = performance.columns.droplevel([0,2])
equally_weighted_returns = performance.reset_index()
.loc[:, ['Period', 'Asset Count']]
.merge(right=returns.reset_index(), right_on='period_count',
                                                left_on='Period', how='left')\
                                        .drop(labels='period_count', axis=1).set_index
                                        .fillna(0)\
                                        .set_index(keys='Period', append=True)\
                                        .apply(lambda x:np.sum(x[x.index != 'Asset Cou
                                                axis=1).to_frame()\
                                        .rename({0:'Equally Weighted'}, axis=1).reset_
                                        .groupby(by='Period').prod()\
                                        .apply(lambda x: (x-1)*100, axis=0)
performance = performance.merge(right=equally_weighted_returns, how='left', right_
performance.head()


# %%
# comparing number of assets used for each optimization
performance.reset_index(level=[1]).loc[:, 'Asset Count'].describe()


# %%
# comparing trajectories of final wealth
((performance.reset_index(level=1, drop=True).sort_values(by='Period')/100 + 1).cu


# %%
final_wealth = ((performance.reset_index(level=1, drop=True).sort_values(by='Perio
final_wealth = final_wealth * 100
final_wealth.name = 'Total Return(%)'
final_wealth = final_wealth.to_frame()
params = dict(float_format='%.2f', bold_rows=True,
              caption="Final Return of portfolios based on minimizing risk measure
final_wealth.sort_values('Total Return(%)',ascending=False)
.to_latex('figures/final_wealth.tex', label='table:final_wealth_table',
                        **params, position='htb')
final_wealth


# %%
fig, ax = plt.subplots(figsize=(16,10))
performance.plot(kind='density', ax=ax)
plt.savefig('./figures/histogram.png')
```

```
# %%
fig, ax = plt.subplots(figsize=(35,10))

performance.plot(kind='box', ax=ax, ylim=(-50, 50), fontsize=20)
ax.grid(False)
ax.yaxis.set_major_formatter(FuncFormatter(lambda y, _: f'{y}%'))
plt.savefig('./figures/boxplot_minimizing_risk.png')


# %%
# summarising return information for all risk measures
summary = performance.describe()
final_sharpe = final_wealth.merge(right=summary.loc['std', :].to_frame(),
                                   right_index=True, left_index=True)\
                                 .apply(lambda x:x['Total Return(%)']/(x['std']
                                 .to_frame()\
                                 .T
final_sharpe.rename({0:'Sharpe Ratio'}, axis=0, inplace=True)
summary = pd.concat([summary, final_wealth.T, final_sharpe])

params = dict(float_format='%.2f',
              bold_rows=True,
              caption="Summary of weekly returns of portfolios created by minimizi

summary.T.sort_values(by=['50%']).iloc[:, 1:].to_latex('./figures/minimizing_risk_s
                 label='table:minimizing_risk_summary',
                 **params)
summary


# %%


# confirm summary only contains numerics
summary_dtypes =  summary.dtypes
assert len(summary_dtypes.loc[summary_dtypes != 'float64'])==0

# ranking and ploting the summary
summary_ranking = summary.drop(labels='count', axis=0).T.drop(labels='std', axis=
summary_ranking.loc[:,'std'] = summary.T.loc[:, 'std'].rank(ascending=False)
summary_ranking = summary_ranking.astype(int)
summary_ranking_inverted = summary_ranking.rank(ascending=False).astype(int)

# plot ranking in heatmap
fig, ax = plt.subplots(figsize=(17,10))
im = ax.imshow(summary_ranking.values, cmap="RdYlGn")

# We want to show all ticks...
ax.set_xticks(np.arange(len(summary_ranking.columns)))
ax.set_yticks(np.arange(len(summary_ranking.index)))
# ... and label them with the respective list entries
```

```
ax.set_xticklabels(summary_ranking.columns)
ax.set_yticklabels(summary_ranking.index)

# Rotate the tick labels and set their alignment.
plt.setp(ax.get_xticklabels(), rotation=45, ha="right",
         rotation_mode="anchor")

# Loop over data dimensions and create text annotations.
heat_map_text_colors = ('black', 'white')
threshold = im.norm(summary_ranking.iloc[:,0].max())/2.
for i in range(len(summary_ranking.index)):
    for j in range(len(summary_ranking.columns)):
        current_rank = summary_ranking_inverted.iloc[i, j]
        color = heat_map_text_colors[int(np.abs(np.ceil(im.norm(current_rank) -thr
        text = ax.text(j, i, current_rank,
                       ha="center", va="center", color=color)

ax.set_title("Performance rank of risk measures across all criteria")
fig.tight_layout()
plt.savefig('./figures/rank_heatmap.png')
plt.show()


# %%
central_measures = summary.loc[['50%', 'mean'], :].T.sort_values(by='50%', axis=0
fig, ax = plt.subplots(figsize=(17,10))
median_comp = ax.barh(y=range(len(central_measures)), width=central_measures.loc[:
                      height=0.4, align='center')
ax.set_xlabel('median return')
ax.set_title('Median Return Comparison')
ax.set_yticks(range(len(central_measures)))
ax.set_yticklabels(central_measures.index)
ax.xaxis.set_major_formatter(FuncFormatter(lambda y, _: f'{y}%'))
ax.grid(False)
for i,j in zip(central_measures.loc[:,'50%'],range(len(central_measures))):
    i_neg = i if i>=0 else -30

    ax.annotate(f'{central_measures.loc[:,"50%"].values[j] :.1f}%',
                xytext=(i_neg,0), xy=(i,j), textcoords='offset points',
                fontsize=10)
plt.savefig('./figures/median_comparison.png')
plt.show()
```

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