Strathmore University

Strathmore Institute of Mathematical Sciences
Bachelor of Business Science Actuarial Science, Financial Engineering & Financial Economics
End of Semester Examination
BSA 2204: Financial Mathematics II

Date: Fri 2nd December 2022   **Time: 2hrs**

Instructions
- This examination consists of FIVE questions.
- Question One carries 30 marks. All other questions carry 20 marks.
- Answer Question ONE (COMPULSORY) and any other TWO questions.

Question 1 [30 marks]

a) State and explain three theories that explain the term structure of interest rates and the shape of the yield curve. [6]

b) A tax-exempt investor purchases Ksh 100,000 nominal of a newly issued 7-year fixed-interest bond that is redeemable at par and pays coupons of 8% per annum half-early in arrears. Calculate:
  (i) The price the investor should pay to obtain an effective annual yield of 9.0%. [2]
  (ii) The running yield on the bond. [1]

c) Distinguish between the terms convexity and duration. [3]

d) Explain the Redington theory of immunization. [3]

e)
   I. The 3-year,5-year and 7-year spot rates used in discounting cashflows are 6%, 5.7% and 5% respectively. The 3-year forward rate from time 4 is 5.2%. Calculate:
     (i) y_4 [1]
     (ii) $f_{5,2}$ [2]
  II. The n-year spot rate of interest is given by:
$$y_n = 0.04 + \frac{n}{1000} \qquad \text{for } n = 1,2,3,4,5$$

Calculate:
(i) The implied 1-year and 2-year forward rates applicable at t=2. [2]
(ii) The 3-year par yield. [2]

f) A company considers that, on average, it will earn interest on its funds at the rate of 4% per annum. However, the investment policy is such that in any one year the yield on the company's funds is equally likely to take any value between 2% and 6%. For both single and annual premium capital redemption policies with terms of 5, 10, 15, 20, and 25 years, and premium £1, find the mean accumulation and the standard deviation of the accumulation at the maturity date. You may ignore all expenses. [4]

g) An insurance company borrows £50 million at an effective interest rate of 9% per annum. The insurance company uses the money to invest in a capital project that pays £6 million per annum payable half-yearly in arrears for 20 years. The income from the project is used to repay the loan. Once the loan has been repaid, the insurance company can earn interest at an effective interest rate of 7% per annum. Calculate the discounted payback period for this investment. [4]

Question 2 [20 marks]

Strathmore Univ. is looking to set up a medical school and offer undergraduate degree programmes to future doctors. As usual the school will be one of its kind in the EA region with state of the art facilities and application of cutting edge research and technology in teaching. Construction is set to begin on 1 January 2023 and will take 3years to complete.

Total construction costs are estimated at KES 2.25 billion and will be spread equally over the 3year construction period. In other words, KES 0.75Bn will be spent each year and will be evenly (continuously) spread through the year.

Alongside construction the university will use its existing channels to start marketing and advertising the upcoming degree, and to hire faculty. Based on past experience it is expected that the first class will have 80 students, each paying a fee of KES 1 million per year. (the full fee for the 5year programme is KES 5Mn and is paid in equal amounts in each year.) Every next class after that should admit 80 students. That way by the fifth year of the programme, the school will receive fee income of KES 400Mn (80 students * 1Mn pa * 5 classes.)

Assuming the following,
▪ No fee increments will ever happen at the school
▪ Operational costs once the school opens are 40% of fee income
▪ Fees are payable approximately halfway through each year (operational costs are also incurred approximately halfway through each year)
▪ Investors financing the project require a compensation of 20% per annum

Calculate
(a) Net present value for the project [9]
(b) Why do you think the net present value is a negative value? What does this indicate? [3]

(c) Do a further calculation to determine if the rate of return is greater than or less than 10%. [4]

(d) Would you invest in this project? Explain why or why not. [2]

Suppose the full fees for new students were increased at a rate of 5% per annum.

(e) What impact would this have on the NPV and IRR? [2]

Question 3 [20 marks]

(a) An investor purchases £100 nominal of a fixed-interest stock, which pays coupons of 7% *pa* half-yearly in arrears. The stock is redeemable at par and can be redeemed at the option of the borrower at any time between 5 and 10 years from the date of issue. The investor is subject to tax at the rate of 40% on income and 25% on capital gains.

   (i) Explain what is meant by the capital gains test.
   In your explanation you should mention the rationale for its calculation. [2]

   (ii) Calculate the maximum price that the investor should pay in order to obtain a net yield of at least 6% pa. [8]

(b) An equity pays half-yearly dividends. A dividend of d per share is due in exactly 3 months' time. Subsequent dividends are expected to grow at a compound rate of g per half-year forever.

   (i) If i denotes the annual effective rate of return on the equity, show that P, the price per share, is given by: [5]

$$P = \frac{d\left(1+i\right)^{\frac{1}{4}}}{\left(1+i\right)^{\frac{1}{2}} - \left(1+g\right)}$$

   (ii) The current price of the share is £3.60, dividend growth is expected to be 2% per half-year and the next dividend payment in 3 months is expected to be 12p. Calculate the expected annual effective rate of return for an investor who purchases the share. [5]

Question 4 [20 marks]

A company has to pay £2,000(10 − t) at the end of year t, for t = 5, 6, 7, 8, 9. It values these liabilities assuming that there will be a constant effective annual rate of interest of 6% *pa*.

(a) Calculate the present value of the company's liabilities. [5]

The company wants to immunise its exposure to the liabilities by investing in two bonds:
☐ Bond A pays coupons of 5% *pa* annually in arrears and is redeemable at par in 15 years' time
☐ Bond B is a zero-coupon bond that is redeemable at par in 5 years' time.

The gross redemption yield on both stocks is the same as the interest rate used to value the liabilities.

(b) Determine the amount that the company should invest in each of the two bonds to ensure that the present value and discounted mean term of the assets are equal to those of the liabilities. [12]

(c) What additional consideration should the company make to immunize its exposure to the liabilities? [3]

Question 5 [20 marks]

(a) Distinguish between a stochastic investment return model and a deterministic one. [2]
(b) In what contexts would it be ideal to use a stochastic investment return model [1]
(c) In what contexts would it be ideal to use a deterministic investment return model [1]

(d) A stochastic model of investment returns assumes that the annual rates of return in different years are independent and identically distributed normal random variables with mean 8% and standard deviation 2%.

Calculate the mean and standard deviation of the accumulated value, at time 2, of an initial investment of £10,000. [7]

(e) The annual rates of return on an investment fund are assumed to be independent and identically distributed. Each year the distribution of $1 + i$ is lognormal with parameters $\mu = 0.07$ and $\sigma^2 = 0.006$, where i is the annual yield on the fund.

Calculate the amount that should be invested in the fund immediately to ensure an accumulated value of at least £500,000 in ten years' time with a probability of 0.99. [9]